What's the point?

A unit of work on decimals with Year three students


Vince Wright
Australian Catholic University
<Vince.Wright@acu.edu.au>

Jacqueline Tjorpatzis
Marymede College
<Jacqueline.Tjorpatzis@marymede.vic.edu.au>

In this article Vince Wright and Jacqui Tjorpatzis share findings from a teaching experiment that involved the implementation of a Year 3 unit of work on decimals. They describe the activities involved, and the challenges and benefits of introducing decimals to young children.

Introduction

In the early 1960s psychologist Jerome Bruner proposed the spiral curriculum. His idea was that learners be exposed to important concepts in a repetitive, cyclic way with increased sophistication as they progressed through schooling. It is easy to look at Bruner's ideas now and see them as common sense. The modern socio-constructivist view is that opportunities to learn in social settings greatly affect a learner's conceptual development. Bruner's most controversial statement was:

> We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. (Bruner, 1960, p. 33)

So what is meant by the phrase "honest form"? To us it means that the concept is presented in a way that is appropriate to what the learner already knows. In this article we share some preliminary results from a teaching experiment. The children were in a Year 3 class at a P–12 College located in the northern suburbs of Melbourne. A unit about decimals was planned for the end of the year as a way to support students to integrate their knowledge about whole number place value and fractions. Research indicates that decimals are difficult to learn and that students often inappropriately apply whole number thinking to decimals (Moody, 2010; Roche & Clarke, 2006; Steinle & Stacey, 2004). This research suggests a need for caution with introducing decimals without adequate foundations. Our Australian National Curriculum first mentions decimals at Year 4:

> Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation. (Australian Curriculum, Assessment, and Reporting Authority, 2013)

In some countries, notably high performing Asian nations, decimals are introduced earlier. For example, in Singapore decimal notation is taught through money in Year 3 and expanded rapidly to three places in Year 4 (Ministry of Education, 2007). So can decimals be learned successfully by students at Year 3?

Interviews at the start of the unit showed that most children in the class preferred to count on and back or use simple part-whole strategies to solve addition and subtraction problems with whole numbers. They were not strong multiplicative thinkers. Their understanding of unit fractions was sound and they could equally partition shapes into halves and quarters. However all students had difficulty reading the height of a child in whole metres and tenths and they could not order decimals correctly. Bruner's idea of honest form presented a significant challenge in this setting.

Choice of materials and context

One of our conjectures was that support from materials and context was likely to be important for students if they were to understand decimals as quantities. We believed that, with experience, students would image the materials and progress to reasoning without physical representations. Stacey, Helme, Archer, and Condon (2001) reported that linear models for representing decimals were more transparent than other models. So we used length as the main attribute and height as the main context, thinking that Year 3 students were keenly interested in information about themselves.

Building lessons around with one physical model (length) and one context (height) proved to be easier than we expected. We used focused variation, particularly playing with the missing information in problems, to change tasks slightly. Straws, pipe cleaners, paper metre tapes donated by Ikea™, Cuisenaire rods and metre lengths of square dowel were our materials of choice. A collection of 1000 straws, pre-bundled into tens then hundreds, allowed us to ask about the relationships between thousands, hundreds, tens and ones. The students' initial ideas were about adding and subtracting amounts between place values but they soon realised that the only thing that worked across all place values was "getting ten of" and the inverse action of partitioning into ten equal parts. When a single straw was cut into ten equal pieces they agreed the pieces should be called tenths.

Figure 1: Mathew's decimal kit made from straws and pipe cleaners

Students then tried to create their own tenths by cutting a straw into ten equal parts. Later we provided a tenths ruler so they could create their own accurate set of ones and tenths for later use.

The context of height allowed us to pose problems involving ordering, addition, and subtraction of decimals. Students found ways to cut the Ikea™ tapes into tenths and measured their partner's height. We created a class list of heights. From the list students predicted the order of height for different groups of students by referring to the numbers and checked out whether the predictions proved correct by having the student line up next to one another. At times tenths were not accurate enough to order students by height.

Figure 2: Students created tenths and measured their heights with paper tapes

'Topping and tailing' students was an engaging context for adding decimals. Student predicted the combined heights and used their straw kits to check if they needed to. They seemed to have no problem relating the whole and tenths of metres to the corresponding pieces in their kits. In one lesson a fictitious party wizard appeared who specialised in cutting children into two pieces by length then putting them back together. The students enjoyed the context and used both subtraction and adding on strategies to solve the problems of the missing part.

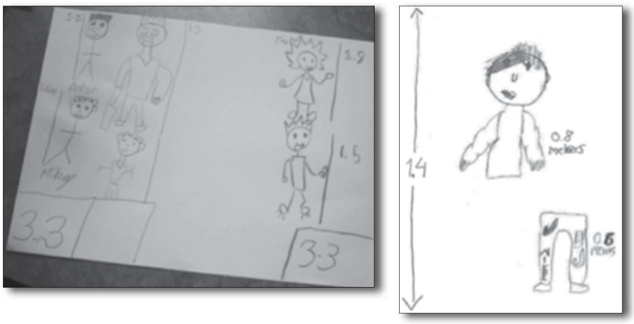

Figure 3: Students' solutions to problems about joining and partitioning heights

Connections to measurement

Another important feature of decimals that became apparent to the students through the use of length was the need for an increased precision of measure. We showed pictures of well-known sports stars with their heights. For example Buddy Franklin, the AFL player, is 1.95 metres tall. Research indicates that students can view the ninety-five in 1.95 as a whole numbers to the right of the decimal point so we were keen that they saw the decimal places as successive partitions by ten. We asked what the students thought the digits in Buddy's height referred to. Many of them had previously noticed centimetres on the Ikea™ tapes but were unsure what centimetre meant. Keen to avoid students building up 1.95 from the individual ticks on the tapes we used the metre long pieces of dowel. The students had already accepted that the orange Cuisenaire rod is one tenth of a metre.

Going back to the place value chart we asked "If one tenth was cut into ten equal parts what would those parts be called?" Several students speculated that the parts would be one centimetre. We asked which rod would be one tenth of one tenth. The white rod was chosen and we predicted how many white rods would fill the one metre length of dowel by laying down only ten white rods alongside it. Some students used skip counting, "ten, twenty, thirty...," while others used multiplication "ten tens are one hundred" to name the white rod as one hundredth. We discussed the connection of cents in one dollar to centimetres in one metre. Using the pieces of dowel and Cuisenaire rods we made a length equal to Buddy's height on the carpet. Then students found their own height to two decimal places.

A possible limitation of using a single attribute like length is that students learn only to think of decimals as lengths. At the end of the two-week unit we were keen to see if they could transfer their understanding to a different attribute, area, and a new operation, division as sharing. We used the context of our favourite chocolate bar that has snap lines for sharing it among friends. The decimat (Roche, 2010) provided a representation of the chocolate bar that allowed for further equal partitioning into tenths, hundredths and thousandths (see Fig. 5). As a class we considered how the bar might be shared between two friends. Students easily established that each person would get five tenths and by thinking about sharing with whole numbers we connected this result to the calculator answer of $1 \div 2 = 0.5$.

Given decimats, scissors and a calculator the students then worked in pairs to explore how a single bar could be partitioned along the 'snap lines' to find equal shares for different numbers of friends; five, ten, four, eight and three. Two things impressed us. The students readily transferred their understanding from length to area and appeared delighted at a new opportunity to do so. Also surprising was their connection of cutting the decimats and using the calculator as validation that they were correct. A few students could even explain why sharing among three resulted in 0.333... since there is "always a piece left over to share".

Practice and new ideas

We found balancing practice and introducing new ideas difficult at times. The students tended to apply their whole number calculation strategies to decimals. For example, students who preferred to count on and back, learned to do so in tenths.

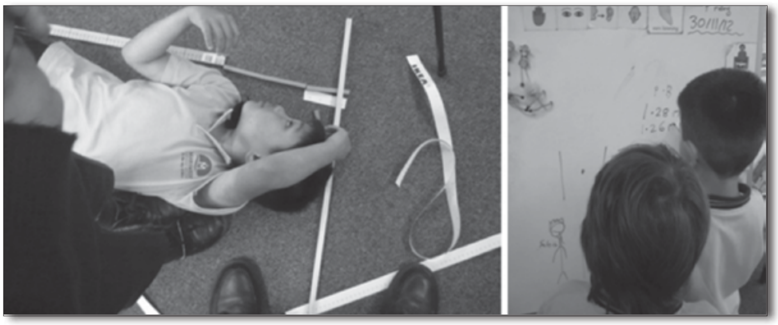

Figure 4: Benny measures his height to the nearest hundredth of a metre while Peter and Carl record their heights

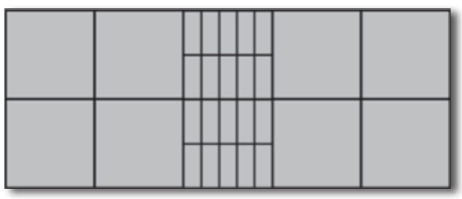

Figure 5: Decimat

Figure 6: Students using decimats and calculators to solve sharing problems

We hoped that their strategies with whole numbers might improve as a pleasant side-effect of working with decimals but this did not seem to happen. Access to physical models gave all students understanding of the quantities involved. However, the ability to use more efficient mental strategies, particularly for multiplication, supported some students in learning the structure of decimals faster than others. The natural tension arose between keeping the class on the same learning path or diversifying instruction though group work. We did a bit of both.

Figure 7: Students playing 3.7 wins

Siegler (2000) pointed out that practice can create new learning. This is especially true if the practice occurs in pursuit of a higher purpose. So we created strategic games in which addition and subtraction of tenths was needed to win. For example, we adapted an old game called 37 to create a game called 3.7 wins (see appendix). Players take turns to move a single counter to adjacent spaces on the board. They cannot jump over a space. Each time the counter moves to a space the number of tenths on that space is added to a shared total. The first player to put the total over 3.7 loses.

Reflection

The use of physical models based on length appeared to present our students with an honest form for learning about decimals and they were all able to engage in the lessons. The students seemed to get a strong sense of quantity from the learning experiences, differentiate between ones and tenths in calculation and transfer their knowledge to equal sharing of area. It was also clear that students' whole number understanding, particularly their use of mental strategies, greatly influenced their ability to anticipate and work with the results of physically acting on the materials.

Given that operating with number properties, without needing to rely on materials, is critical for more advanced work (Pirie & Kieren, 1994) it could be argued that it is better to leave decimal instruction until later class levels. We accept that argument to some extent. In our end-of-unit discussion Jacqui pointed out, "Decimals are part of their everyday life. Why shouldn't children learn about them at this age?" This class of Year 3 students did come to understand aspects of decimal place value. For some students their understanding was equipment-reliant and limited to tenths while few students seemed to generalise the system in abstract ways to three places. Bruner's 'honest form' carries responsibility to introduce concepts in ways that are appropriate developmentally, not just accessible. Strong decimal understanding is based on knowledge about 'nested' place value and equivalence with fractions, and fluent mental calculation strategies with whole numbers (Wright, 2004) and students develop this knowledge through opportunities to learn. The idea that class level, by itself, is an adequate indicator of this readiness is debatable but year 3 may be too early for many, if not most. We believe that the authentic learning experiences about decimals we provided would also be appropriate and potentially even more successful for older students with stronger foundation knowledge.

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3.7 Wins

You need: A partner, one transparent counter

To play: Start with Player A putting the counter on 5 tenths. The score is now 0.5.
Write the score down.Players take turns moving the counter along a blue line to an adjacent star.
The number on the star is added to the total and the new total is written down. A player wins if they make the total 3.7 or, in some other way, force their opponent to go over 3.7.